INDIAN SCHOOL MUSCAT
SECOND PRELIMINARY EXAMINATION
MATHEMATICS

CLASS: XII   Sub. Code: 041   Time Allotted: 3 Hrs

17.02.2019   Max. Marks: 100

General Instructions:

1. The question paper consists of four sections
   Q(1-4) are of 1 mark each, Q(5-12) are of 2 marks each, Q(13-23) are of 4 marks each and Q(24-29) are of 6 marks each.
2. There is no overall choice, all questions are compulsory however where internal choices are given students are expected to attempt any one of the two choices.

SECTION A   (4 x 1 = 4)

1. If $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$ then find the value of $(\vec{a} \times \hat{i}).(\vec{a} \times \hat{j}) + xy$

   OR

   Find the value of p so that the vectors $3\hat{i} - \hat{j} - 5\hat{k}$ and $2\hat{i} + 3\hat{j} - p\hat{k}$ are perpendicular.

2. For what value of 'β' the matrix $\begin{bmatrix} \beta & 3 \\ 2 & 4 \end{bmatrix}$ has no inverse.

3. Determine the order and degree of the equation.

   $$\left(\frac{ds}{dt}\right)^2 + 3s\left(\frac{d^2s}{dt^2}\right)^2 = 0.$$

4. Differentiate $\log(\tan\sqrt{x^2 + 1})$ with respect to x.

SECTION B   (8 x 2 = 16)

5. If $A = \begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix}$, verify that $A - A^T$ is a skew-symmetric matrix.

6. A bag contains 8 black and 7 white balls. Two balls are drawn from the bag one after the other without replacement. What is the probability that both drawn balls are black?

7. Evaluate : $\int_1^2 \frac{\sqrt{3-x}}{\sqrt{x} + \sqrt{3-x}} dx$ **OR** $\int \frac{sec^2(2tan^{-1}x)}{1+x^2} dx$

8. Find the differential equation of the family of curves $y = \frac{c}{x} + Dx$

9. Evaluate : $\int e^x \left(-\frac{2}{x^3} + \frac{1}{x^2}\right) dx$

10. Let * be a binary operation on A where $A = Q \times Q$. Let * be defined as $(a, b) * (c, d) = (ac, b + ad)$ for $(a, b), (c, d) \in A$. Find the identity element in A.

11. For 6 trials of an experiment, let X be a binomial variate which satisfies the relation $9 P(X = 4) = P(X = 2)$. Find the probability of success.

OR

The probability that atleast one of the events A or B occurs is $\frac{3}{5}$. If A and B occur simultaneously with probability $\frac{1}{5}$. Find $P(\overline{A}) + P(\overline{B})$

12. Find a vector $\vec{a}$ of magnitude $5\sqrt{2}$, making an angle of $\frac{\pi}{4}$ with X- axis, $\frac{\pi}{2}$ with Y- axis and an acute angle β with Z – axis.

OR

If $\vec{a} = 4\hat{\imath} + 2\hat{\jmath} - \hat{k}$ and $\vec{b} = 5\hat{\imath} + 2\hat{\jmath} - 3\hat{k}$, find the angle between $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$.

SECTION C   (11 x 4 = 44)

13. Prove that the relation R in the set $A = \{1,2,3,4,5\}$ given by $R = \{(a,b) : |a - b| \text{ is even}\}$ is an equivalence relation.

OR

If $f(x) = \frac{4x+3}{6x-4}$, $x \neq \frac{2}{3}$, show that $(fof)(x) = x$ for all $x \in R - \{\frac{2}{3}\}$. What is the inverse of f ?

14. Solve the differential equation: $x^2 dy + (xy + y^2)dx = 0$; given that $y = 1$, when $x = 1$.

OR

Solve the differential equation $(y \cot x - \cos^2 x) dx + dy = 0$

15. If $a \neq b \neq c$ and $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix} = 0$ then show that $1 + abc = 0$

OR

Using properties of determinants show that $\begin{vmatrix} a^2 + 1 & ab & ac \\ ab & b^2 + 1 & bc \\ ac & bc & c^2 + 1 \end{vmatrix} = a^2 + b^2 + c^2 + 1$

16. If the following function, $f(x) = \begin{cases} x^2 + ax + b, & 0 \leq x < 2 \\ 3x + 2, & 2 \leq x \leq 4 \\ 2ax + 5b, & 4 < x \leq 8 \end{cases}$ continuous on $[0,8]$, find the value of a and b.

17. Find $\dfrac{dy}{dx}$ where $y = x^{\tan x} + \log(x + \sqrt{1 + x^2})$

18. For any three vectors $\vec{a}, \vec{b}, \vec{c}$ show that $[\vec{a} \cdot [(\vec{b} + \vec{c}) \times (\vec{a} + 2\vec{b} + 3\vec{c})]] = [\vec{a}, \vec{b}, \vec{c}]$.

19. Solve for x : $\tan^{-1} 2x + \tan^{-1} 3x = \dfrac{\pi}{4}$.

20. Find the intervals in which the function f defined by $f(x) = x^3 + \frac{1}{x^3}$ is a) increasing   b) decreasing.

21. Evaluate : $\int \frac{3x+1}{(x^2+4)(x-1)} dx$

22. Find the shortest distance between the lines
$\vec{r} = 3\hat{\imath} + 5\hat{\jmath} + 7\hat{k} + \lambda(\hat{\imath} + 2\hat{\jmath} + 7\hat{k})$ and $\vec{r} = -\hat{\imath} - \hat{\jmath} + \hat{k} + \mu(7\hat{\imath} - \hat{\jmath} + \hat{k})$

23. Evaluate : $\int_0^1 \cot^{-1}(1 - x + x^2) dx$

SECTION D (6 x 6 = 36)

24. Find the point of local maxima and local minima of the function f(x) = sin x – cos x, 0 < x < 2π. Also find the local maximum and local minimum values.

25. A company manufacture two types of lamps say A and B. Both lamps go through a cutter and then a finisher. Lamp A requires 2 hours of the cutter's time and 1 hours of the finisher's time. Lamp B requires 1 hour of cutter's and 2 hours of finisher's time. The cutter has 100 hours and finisher has 80 hours of time available each month. Profit on one lamp A is Rs. 7 and on one lamp B is Rs. 13. Assuming that he can sell all that he produces, formulate a LPP and solve it graphically to find out how many of each type of lamps should be manufactured to obtain maximum profit?

26. Using matrix method find the inverse of matrix A where $A = \begin{bmatrix} 2 & 1 & 3 \\ 3 & -2 & -1 \\ 3 & 1 & -2 \end{bmatrix}$.
Hence solve the following system of linear equations:
2x +3 y + 3 z = 5 ; x – 2y + z = – 4 ; 3 x – y –2 z = 3

27. In answering a question on a multiple choice, a student either knows the answer or guesses. Let $\frac{3}{4}$ be the probability that he knows the answer and $\frac{1}{4}$ be the probability that he guesses. Assuming that a student who guesses at the answer will be correct.
What is the probability that the student knows the answer, given that he answered it correctly?
OR
From a lot of 15 bulbs which include 5 defectives, a sample of 4 bulbs is drawn one by one with replacement. Find the probability distribution of number of defective bulbs. Also find the mean and variance of this distribution.

28. Find the area of the smaller region bounded by the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and the line $\frac{x}{3} + \frac{y}{2} = 1$.
OR
Find the ratio of the areas into which the curve $y^2 = 6x$ divides the region bounded by $x^2 + y^2 = 16$.

29. Find the distance of the point (- 2 , 3, - 4) from the line $\frac{x+2}{3} = \frac{2y+3}{4} = \frac{3z+4}{5}$ measured parallel to the plane 4x +12y – 3z +1 = 0
OR
Find the vector and the Cartesian equation of the line passing through (1, 2, 3) and parallel to the planes x – y + 2z = 5 and 3x + y + z = 6.

End of the Question Paper

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2. Differentiate, $\log(\tan\sqrt{x^2+1})$ with respect to x.

3. If $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$ then find the value of $(\vec{a} \times \hat{i}).(\vec{a} \times \hat{j}) + xy$

OR

Find the value of p so that the vectors $3\hat{i} - \hat{j} - 5\hat{k}$ and $2\hat{i} + 3\hat{j} - p\hat{k}$ are perpendicular.

4. For what value of 'k' the matrix $\begin{bmatrix} \beta & 3 \\ 2 & 4 \end{bmatrix}$ has no inverse.

SECTION B   (8 x 2 = 16)

5. Let * be a binary operation on A where A = Q X Q. Let * be defined as (a, b) * (c, d) = (ac, b + ad) for (a, b), (c, d) $\in A$. Find the identity element in A.

6. An urn contains 8 black and 7 white balls. Two balls are drawn from the urn one after the other without replacement. What is the probability that both drawn balls are black?

7. Find a vector $\vec{a}$ of magnitude $5\sqrt{2}$,making an angle of $\frac{\pi}{4}$ with X- axis, $\frac{\pi}{2}$ with Y- axis and an acute angle β with Z – axis.

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8. Find the differential equation of the family of curves $y = \dfrac{c}{x} + Dx$

9. Evaluate : $\int e^x \left(\dfrac{1}{x^2} - \dfrac{2}{x^3} \right) dx$

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13. Solve for x : $\tan^{-1} 2x + \tan^{-1} 3x = \dfrac{\pi}{4}$.

14. If $x \neq y \neq z$ and $\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$, then show that $1 + xyz = 0$

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